APEC 8001: Problem Set 1 Answer Key

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1. Consider a choice problem with choice set $X = \{x, y, z\}$. For each of the following choice structures say whether WARP is satisfied and whether there exists a rational preference relation that rationalizes $\mathcal{C}(.)$ relative to its $\mathcal{B}$. If such rationalization is possible, write it down. Comment on your results.

   (a) $(\mathcal{B}', \mathcal{C}(.))$, where $\mathcal{B}' = \{\{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $\mathcal{C}(\{x, y\}) = \{x\}, \mathcal{C}(\{y, z\}) = \{y\}, \mathcal{C}(\{x, z\}) = \{z\}, \mathcal{C}(\{x\}) = \{x\}, \mathcal{C}(\{y\}) = \{y\}, \mathcal{C}(\{x\}) = \{z\}$

   Sol.
   $\mathcal{C}(\{x, y\}) = \{x\}$ reveals $x \succ y$
   $\mathcal{C}(\{y, z\}) = \{y\}$ reveals $y \succ z$
   $\mathcal{C}(\{x, z\}) = \{z\}$ reveals $z \succ x$
   Then, W.A.R.P. is satisfied but $(\mathcal{B}', \mathcal{C}(.))$ is not rationalizable because preferences are not transitive.

   (b) $(\mathcal{B}'', \mathcal{C}(.))$, where $\mathcal{B}'' = \{\{x, y, z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $\mathcal{C}(\{x, y, z\}) = \{x\}, \mathcal{C}(\{x, y\}) = \{x\}, \mathcal{C}(\{y, z\}) = \{z\}, \mathcal{C}(\{x, z\}) = \{z\}, \mathcal{C}(\{x\}) = \{x\}, \mathcal{C}(\{y\}) = \{y\}, \mathcal{C}(\{z\}) = \{z\}$

   Sol.
   $\mathcal{C}(\{x, y, z\}) = \{x\}$ reveals $x \succ y$ and $x \succ z$
   $\mathcal{C}(\{x, y\}) = \{x\}$ reveals $x \succ y$
   $\mathcal{C}(\{y, z\}) = \{z\}$ reveals $z \succ y$
   $\mathcal{C}(\{x, z\}) = \{z\}$ reveals $z \succ x$
   Then, W.A.R.P. is not satisfied because $x \succ z$ and $z \succ x$ are satisfied at the same time. $(\mathcal{B}', \mathcal{C}(.))$ is not rationalizable.

(c) $(\mathcal{B}''', \mathcal{C}(.))$, where $\mathcal{B}''' = \{\{x, y, z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $\mathcal{C}(\{x, y, z\}) = \{x\}, \mathcal{C}(\{x, y\}) = \{x\}, \mathcal{C}(\{y, z\}) = \{y\}, \mathcal{C}(\{x, z\}) = \{x\}, \mathcal{C}(\{x\}) = \{x\}, \mathcal{C}(\{y\}) = \{y\}, \mathcal{C}(\{z\}) = \{z\}$

Sol.

$\mathcal{C}(\{x, y, z\}) = \{x\}$ reveals $x \succ y$ and $x \succ z$

$\mathcal{C}(\{x, y\}) = \{x\}$ reveals $x \succ y$

$\mathcal{C}(\{y, z\}) = \{y\}$ reveals $y \succ z$

$\mathcal{C}(\{x, z\}) = \{x\}$ reveals $x \succ z$

Then, W.A.R.P. is satisfied and $(\mathcal{B}', \mathcal{C}(.))$ is rationalizable.

2. A consumer chooses bundles of two goods. In one situation $(p_1, p_2) = (1, 1), w = 1$ and $(x_1, x_2) = (0.5, 0.5)$. At the new situation, $(p_1^{'}, p_2^{'}) = (2, p_2^{'})$ and $(x_1^{'}, x_2^{'}) = (0.75, 0.25)$. Assuming her demand satisfies Walras' law, what are the values of $p_2^{'}$ and w' that make her choices compatible with WARP?

Sol.

If her preferences are compatible with WARP and they satisfy Walras' law, then if $p.x(p', w') \leq w$ and $x(p', w') \neq x(p, w)$, then $p'.x(p, w) > w'$.

The first part holds because $(1 \times 0.75) + (1 \times 0.25) = 1$. Also, both, the initial bundle and the second one are different. Then, it must be true that $(2 \times 0.5) + 0.5p_2^{'} > w'$ (with strict inequality!).

$$1 + 0.5p_2^{'} > w' \tag{1}$$

Because Walras' law must hold with the second bundle, we also know that $(2 \times 0.75) + 0.25 \times p_2^{'} \leq w'$.

$$1.5 + 0.25p_2' \leq w' \tag{2}$$

Combining equations 1 and 2, we obtain that $p_2' > 2$. Notice from equation 2 that $p_2' \leq \dfrac{w' - 1.5}{0.25}$. Then, $w' > 2$. So, the values that make her choices compatible with WARP are $p_2' \in (2, \infty)$ and $w \in (2, \infty)$. Notice that these are open intervals.

3. The example in page 14 of lecture 1 satisfies the week axiom. Show that adding the

budget set $\{x, y, z\}$ to $\mathcal{B}$ implies that the choices made in that example violate the weak axiom.

The example in p.14 reveal that $x \succ y, y \succ z$, and $z \succ x$. If we include $\{x, y, z\}$ in the budget set:

- If $C(\{x, y, z\}) = \{x\}$ then, $x \succ y$ and $x \succ z$
- If $C(\{x, y, z\}) = \{y\}$ then, $y \succ x$ and $y \succ z$
- If $C(\{x, y, z\}) = \{z\}$ then, $z \succ x$ and $z \succ y$
- If $C(\{x, y, z\}) = \{x, y\}$ then, $x \succ z$, $y \succ z$ and $x \sim y$
- If $C(\{x, y, z\}) = \{x, z\}$ then, $x \succ y$, $z \succ y$ and $x \sim z$
- If $C(\{x, y, z\}) = \{y, z\}$ then, $y \succ x$, $y \succ z$ and $y \sim z$
- If $C(\{x, y, z\}) = \{x, y, z\}$ then, $x \sim y \sim z$

Then every choice made when including $\{x, y, z\}$ is incompatible with the original three choices.