Problem Set 2 - Answer Key

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1. Show whether lexicographic preferences are complete, transitive, (strongly) monotone, and (strictly) convex.

   Sol.

   1. Completeness means that for all x and y, either $x \succeq y$ or $y \succeq x$. Suppose that without loss of generality only two goods, $x_1 \geq y_1$, then $x \succ y$. If $x_1 = y_1$ and $x_2 > y_2$, then $x \succ y$; if $x_1 = y_1$ and $x_2 = y_2$, then $x \sim y$. Then lexicographic preferences are complete.

   2. To show transitivity, suppose that $x \succeq y \succeq z$. If $x_1 > y_1 > z_1$, transitivity holds, since $x_1 > z_1$ imply $x \succ z$. If $x1 = y_1 > z_1$, again transitivity holds, since $x_1 > z_1$ imply $x \succ z$. Suppose $x_1 = y_1 = z_1$. Then if $x_2 > y_2 \geq z_2$ or $x_2 = y_2 > z2$, we have $x \succ z$. The only remaining case is $x \sim y \sim z$, for which the transitivity relationship is trivial

   3. Strong monotonicity: suppose $x \geq y$ and $x \neq y$. Then either $x_1 > y_1$, or $x_1 = y_1$ and $x_2 > y_2$, and in either case $x \succ y$.

   4.To show convexity, suppose that $y \succeq x$ and $z \succeq x$. This implies that y_1 and z_1 are at least as good as x_1, so that if either y_1 or z_1 is strictly greater than x_1, we have $\alpha z + (1\alpha)y \succ x$. Suppose then that $y_1 = z_1 = x_1$. Then since $y \succeq x$ and $z \succeq x$, we must have $y_2, z_2 > x_2$, and $\alpha z + (1 - \alpha)y \succ x$.

2. State and prove whether the following statements are true or false.

   - Any homothetic, continuous, and monotonic preference relation on the commodity bundle space can be represented by a continuous utility function that is homogeneous of degree one.

     Sol.

     True. Continuous and monotonic preferences imply that if $x \sim y$, then $U(x) = U(y) \Rightarrow \alpha U(x) = \alpha U(y)$. On the other hand it also implies that if $\alpha x \sim \alpha y$, then $U(\alpha x) = U(\alpha y)$.

     Preferences are homothetic if $x \sim y$ implies $\alpha x \sim \alpha y$. Then it must be true that $U(\alpha x) = U(\alpha y) = \alpha U(x) = \alpha U(y)$

   - If preferences are transitive, convex, and continuous, they must be locally non-satiated

     Sol.

     False. One example is the utility function in 4.(c)

- If the utility function is homothetic, the marginal utility of income is independent of prices and depends only on income

  Sol.

  False. First, let's show that x is HD1 in w by contradiction. We know that $x(p, \alpha w)$ and $\alpha x(p, w)$ are feasible on αw. But $x(p, \alpha w)$ is the optimal choice. Then, it must be true that:

$$U(x(p, \alpha w)) > U(\alpha x(p, w))$$
$$\frac{U(x(p, \alpha w))}{\alpha} > \frac{U(\alpha x(p, w))}{\alpha}$$
$$U\left(\frac{x(p, \alpha w)}{\alpha}\right) > U\left(\frac{\alpha x(p, w)}{\alpha}\right)$$
$$U\left(\frac{x(p, \alpha w)}{\alpha}\right) > U\left(x(p, w)\right)$$

  The third line is because when preferences are homothetic, then U is HD1. The latter line implies a contradiction because the term on the right should be the optimum, yet, there is another bundle that is affordable and that reports more utility. So, it must be true that $x(p, \alpha w) = \alpha x(p, w)$ if preferences are homothetic.

  If that is true, then $x(p, w) = wx(p)$. Then, the marginal utility of income can be written as follows

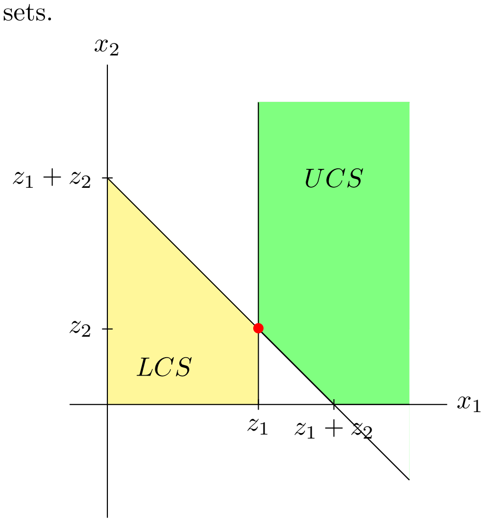
$$\frac{\partial v(x(p, w))}{\partial w} = \frac{\partial v(wx(p))}{\partial w} = v(x(p))$$

- Lexicographic preferences are homothetic.

  Sol.

  True. The proof is trivial. If $x_1 > y_1$, then $x \succ y . \forall \alpha > 0$, then it is also true that $\alpha x_1 > \alpha y_1$, which implies $\alpha x \succ \alpha y$. Similarly, if $x_1 = y$, then $x \sim y$, and $\alpha x_1 = \alpha y$, then $\alpha x \sim \alpha y$

3. A consumer has preferences $\succeq$ for goods 1 and 2 defined by: $(x_1, x_2) \succeq (z_1, z_2) \iff x_1 \geq z_1$ and $x_1 + x_2 \geq z_1 + z_2$. Show whether these preferences are complete, reflexive, transitive, locally non-satiable, and convex. Justify your answer and draw the indifference, upper contour, and lower contour sets.



Sol.

These preference are not complete (because there are areas without color) but they are reflexive, transitive, locally non satiated, convex but not continuous. The upper contour set is the green area, the lower contour set is the yellow area, the indifference "curve" is the red point.

4. Sketch indifference curves for the following utility functions. If it helps, you can do a transformation of the utility function before sketching the indifference curves. In each case, say whether the underlying preference ordering is convex or strictly convex

   (a) $U(x_1, x_2) = e^{min\{x_1,x_2\}^2}$

   (b) $U(x_1, x_2) = \frac{1}{2}x_1^2 + \ln x_2$

   (c) $U(x_1, x_2) = 4 - [(x_1 - 2)^2 + (x_2 - 3)^2]$

   (d) $U(x_1, x_2) = \min\{\sqrt{x_1 x_2}, x_2\}$