Problem Set 4 - Answer Key

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1. One way to compare budget sets is by using the indirect preferences that involve comparing $x(p, w)$ and $x(p', w)$. There are other two approaches to making such comparison: CV, and EV. Solve the following exercises regarding a consumer in a two-commodity world with a utility function u.

   (a) For the case of preferences represented by $u(x_1, x_2) = x_1 + x_2$, calculate the two consumer surplus measures.

   You can do two procedures to calculate the expenditure functions: 1.You have to calculate the marshalian demands, then the indirect utility function and solve for w. 2. Instead of maximizing the utility function, you can minimize the budget subject to the utility function. In either case you should arrive to:

   $v(p, w) = \dfrac{w}{\min\{p_1, p_2\}}$

   $e(p, u) = u \min\{p_1, p_2\}$

   Let $p = \{p_1, p_2\}$ and $p' = \{p'_1, p_2\}$. Also $v = v(p, w)$ and $v' = v(p', w)$ then:

   $EV = e(p, v') - e(p, v) = v' \min\{p_1, p_2\} - w = \dfrac{\min\{p_1, p_2\}}{\min\{p'_1, p_2\}} w - w$

   $CV = e(p', v') - e(p', v) = w - v \min\{p'_1, p_2\} = w - w\dfrac{\min\{p'_1, p_2\}}{\min\{p_1, p_2\}}$

   (b) Show that the first good is a normal good (the demand is increasing with wealth). What is the relation of the two measures to the "area below the demand function" (which is a standard third definition of consumer surplus)?

   You can show the first part by taking the derivative of the marshallian demands w.r.t. wealth. I'll skip this part because it is trivial. To show the second part of the question, let $A = \dfrac{\min\{p_1, p_2\}}{\min\{p'_1, p_2\}}$. Then:

   $EV = w(A - 1)$

   $CV = w\{1 - \dfrac{1}{A}\} = w\dfrac{A-1}{A} = \dfrac{w}{A} EV$

(c) Explain why the two measures are identical if the individual has quasilinear preferences in the second commodity and in a domain where the two commodities are consumed in positive quantities.

Notice that with quasilinear preferences, increases in wealth will not change the consumption of the good. Therefore there's no wealth effect.

2. Consider a two good world, where preferences are strictly quasi-concave and the expenditure function is given by: $e(p_1, p_2, u)$. Let p^0, p^1, p^2 represent three different price vectors, and assume (when needed) that income is constant at w. Let $u^0 = V(p^0, w)$ where V is the indirect utility function for the same preferences.

   (a) Define a price index, relative to the initial situation, by: $I(p^0, p^i, u^0) = \dfrac{e(p^i, u^0)}{e(p^0, u^0)}$. If $I > 1$ what, if anything, can we conclude about the value of $V(p^i, w)$ compared to u^0?

   (b) If preferences are homothetic, does the value of the price index depend on u^0?

   (c) Suppose preferences are quasi-linear, with the income elasticity of demand for good 1 being zero (at an interior solution). How does an increase in the base utility (u^0) - which, given prices, corresponds to an increase in w affect the value of this index?

   (d) What is the relationship between $CV(p^0, p^1, w)$ and $EV(p^0, p^1, w)$ if $p_1^0 = p_1^1$, and $p_2^0 \neq p_2^1$?

   (e) Let p^1, p^2 be two distinct price vectors and assume $CV(p^0, p^1, w) > CV(p^0, p^2, w)$. Can we conclude that for any rational and continuous preferences $V(p^1, w) > V(p^2, w)$? Hint: Consider some specific functional forms for preferences.

3. Suppose there is an economy in which there are n people, $i = 1, ..., n$. Person i has a utility function of the form

$$u_i(x_1, x_2) = A_i \ln(x_1 + b_i) + \ln(x_2 - 1)$$

What restrictions, if any, do we need to put on the parameters A_i and b_i so that aggregate demands for goods 1 and 2 are determined by prices and the sum of incomes and do not depend on the distribution of income? Relate your answer to the Gorman polar form.

4. Consider an "augmented" linear expenditure system (LES). The utility function with on one consumption good, x, and leisure l is:

$$U(x, l) = \alpha \ln(l - \bar{l}) + (1 - \alpha) \ln(x - \bar{x})$$

where $\bar{l}$ denotes committed leisure and $\bar{x}$ denotes committed consumption. The expenditure function for the above LES takes the form:

$$e(u, p, w) = w\bar{l} + p\bar{x} + ub(p, w)$$

where $b(p, w) = w^{\alpha} p^{1-\alpha}$. The indirect utility function takes the form

$$v(w,p,I) = \frac{[I - (w\bar{l} + p\bar{x})]}{b(p,w)}$$

where I is full income.

(a) Derive the Hicksian demand for leisure, denoted l^h

Using Shephard's lemma: $\frac{\partial e(p,u)}{\partial w} = l^h = \bar{l} + u\alpha w^{\alpha-1}p^{1-\alpha}$

(b) Derive the Marshallian demand for leisure, denoted l^m, using l^h.

Plugging $v(w,p,I)$ in the hicksian demand: $l^m = \frac{\alpha I + (1-\alpha)w\bar{l} - \alpha p\bar{x}}{w}$

(c) Preferences described by the above utility function are separable. Show that:

$$\frac{\partial l^h}{\partial p} = S_{lx} = \mu \frac{\partial l^m}{\partial I}\frac{\partial x^m}{\partial I}$$

where μ is a constant which does not depend on l or x.

Applying the same procedure that we used to find l^m, we can easily show that:

$$x^h = \bar{x} + (1-\alpha)uw^{\alpha}p^{-\alpha}$$
$$x^m = \frac{(1-\alpha)I - (1-\alpha)w\bar{l} + \alpha p\bar{x}}{p}$$

Taking the derivatives:

$$\frac{\partial l^h}{\partial p} = (1-\alpha)u\alpha w^{\alpha-1}p^{-\alpha}$$
$$\frac{\partial l^m}{\partial I} = \frac{\alpha}{w}$$
$$\frac{\partial x^m}{\partial I} = \frac{1-\alpha}{p}$$

Plugging in the equation given:

$$(1-\alpha)u\alpha w^{\alpha-1}p^{-\alpha} = \mu\frac{\alpha}{w}\frac{1-\alpha}{p}$$
$$\mu = uw^{\alpha}p^{1-\alpha}$$
$$\mu = I - w\bar{l} - p\bar{x} \because u = v(w,p,I])$$

(d) What does this imply about substitutability between leisure and consumption in thie model? (A single sentence will suffice)

It implies leisure and consumption are substitutes.

(e) How could you introduce individual heterogeneity into this model to derive an estimable labor supply equation? Assume that everyone works and that hours worked is $T - l^m$ Multiple answers are possible. One possibility is to introduce heterogeneity in the preferences for labor and consumption, i.e., $U_i(x,l) = \alpha_i \ln(l_i - \bar{l}) + (1-\alpha_i)\ln(x_i - \bar{x})$

(f) Consider a theoretical model of family utility represented by a single LES utility function, with three arguments: consumption (assumed to be a jointly consumed good), partner a's leisure and partner b's leisure. If you believed that a and b's leisure might be complements, how would you modify the basic LES utility function?

One option would be: $U(l_1, l_2, x) = (min\{l_1 - \bar{l_1}, l_2 - \bar{l_2}\})^{\alpha}x^{1-\alpha}$