APEC 8001: Recitation notes 1*

Julieth Santamaria

September 7, 2017

1 Preference relations

The preference relation is denoted by $\succeq$. Compares two alternatives:

- $x \succeq y \iff x$ is at least as good as y
- $x \succ y \iff x \succeq y$ but not $y \succeq x$
- $x \sim y \iff x \succeq y$ and $y \succeq x$

Preferences are rational if

- Complete: $\forall x, y \in X,\ x \succeq y,\ y \succeq x$, or both
- Transitive: $\forall x, y, z \in X,\ \ x \succeq y,\ y \succeq z$, then $x \succeq z$

If $\succeq$ is rational, then

- $\succ$ is
  - Irreflexive $\iff x \succ x$ does not hold
  - Transitive $\iff x \succ y,\ y \succ z$, then $x \succ z$
- $\sim$ is
  - Reflexive $\iff x \sim x$ holds
  - Transitive $\iff x \sim y,\ y \sim z$, then $x \sim z$
  - Symmetric $\iff x \sim y$ and $y \sim x$

Exercises

1. MWG, Question 1.B.1. Prove that if $\succeq$ is rational, then if $x \succ y \succeq x$, then $x \succ z$
2. Prove that $\succ$ is irreflexive
3. Prove that $\succ$ is transitive

*Based on lecture notes and other material by Paul Glewwe. Some examples and exercises are from Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). Microeconomic theory. New York: Oxford university press.

2 Choice rules

Choice behaviour is represented by a choice structure that has two components:

- $\mathcal{B}$: a set of nonempty subsets of X. Every element of $\mathcal{B}$ is a set $B \subset X$
- $C(.)$ is a choice rule that assigns a nonempty set of chosen elements of B, i.e. $C(B)\ subsetB$ for every budget set in $\mathcal{B}$

W.A.R.P Def.: The choice structure $(\mathcal{B}, C(.))$ satisfies the weak axiom of revealed preference if for some $B \in \mathcal{B}$, with $x, y \in B$, we have $x \in C(B)$, then for any $B' \in \mathcal{B}$, with $x, y \in B'$, and $y \in C(B')$, we must also have $x \in C(B')$

Note: Rational preferences generate a choice structure that satisfies WARP. However, WARP itself is not sufficient to guarantee the existence of rationalizing preference relations.

Exercise

1. Consider the choice structure $(\mathcal{B}, C(.))$ with $\mathcal{B} = (\{x, y\}, \{x, y, z\})$ and $C(\{x, y\}) = \{x\}$. Show that if $(\mathcal{B}, C(.))$ satisfies W.A.R.P., then we must have $C(\{x, y, z\}) = \{x\}$, $= \{z\}$, or $= \{x, z\}$

3 Utility function

Utility function $u : X \to \mathbb{R}$ represents $\succeq$ on X iff $x \succeq y \iff u(x) \geq u(y)$

- u represents preferences only if $\succeq$ is rational
- If $X \subseteq \mathbb{R}^n$, then $\succeq$ can be represented by a continuous utility function iff $\succeq$ is continuous
- u is ordinal
- u is not unique

Exercise

1. Prove $u(.)$ is ordinal