# APEC 8002: Recitation 2[1]

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Suppose a producer has two manufacturing plants (denoted by A and B) that produce a single identical output ($q^j$ for $j = A, B$) using the same two inputs ($z_1^j$ and $z_2^j$ for $j = A, B$) with production possibility sets that are nonempty, closed, satisfy weak free disposal, and satisfy strict convexity. The jth plant cost function is:

$$C^j = \frac{r_1 r_2}{\left(r_1^{\alpha^j} + r_2^{\alpha^j}\right)^{1/\alpha^j}} {q^j}^2$$

Where $r_1 > 0$ and $r_2 > 0$ are price of the first and second input, and $\alpha^j > 0$

a) Find the producer's profit maximizing supplies assuming the competitive price of output is p>0.

$$\max_{q^A \geq 0; q^B \geq 0} \pi = p(q^A + q^B) - \frac{r_1 r_2}{\left(r_1^{\alpha^A} + r_2^{\alpha^A}\right)^{\frac{1}{\alpha^A}}} {q^A}^2 - \frac{r_1 r_2}{\left(r_1^{\alpha^B} + r_2^{\alpha^B}\right)^{1/\alpha^B}} {q^B}^2$$

$$q^{A*} = \frac{P\left(r_1^{\alpha^A} + r_2^{\alpha^A}\right)^{1/\alpha^A}}{2 r_1 r_2}$$

$$q^{B*} = \frac{P\left(r_1^{\alpha^B} + r_2^{\alpha^B}\right)^{1/\alpha^B}}{2 r_1 r_2}$$

b) What are the shares of output across plants? What parameters do these shares depend on and what is the economic intuition of this result?

$$\theta^A = \frac{\left(r_1^{\alpha^A} + r_2^{\alpha^A}\right)^{\frac{1}{\alpha^A}}}{\left(r_1^{\alpha^A} + r_2^{\alpha^A}\right)^{1/\alpha^A} + \left(r_1^{\alpha^B} + r_2^{\alpha^B}\right)^{1/\alpha^B}}$$

$$\theta^A = \frac{\left(r_1^{\alpha^B} + r_2^{\alpha^B}\right)^{1/\alpha^B}}{\left(r_1^{\alpha^A} + r_2^{\alpha^A}\right)^{1/\alpha^A} + \left(r_1^{\alpha^B} + r_2^{\alpha^B}\right)^{1/\alpha^B}}$$

Notice they don't depend on prices

[1] Based on lecture notes and material from previous years.

c) Derive the producer's profit function and then use this profit function to derive its aggregate unconditional supply and unconditional input demands.

Plug $q^{A*}$ and $q^{B*}$ in the profit function ($\pi$). To find the unconditional supply use the identity $q = \frac{\partial \pi}{\partial P}$. For the unconditional input demands use $z_i = -\frac{\partial \pi}{\partial r_i}, \forall i = 1,2$