APEC 8001: Recitation notes 2*

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September 19, 2017

1 Preferences and utility: Definitions

- Monotonicity: The preference relation $\succeq$ on X is monotone iff $x \in X$ and $y >> x$ imply that $y \succ x$
- Strong monotonicity: The preference relation $\succeq$ on X is strongly monotone iff $x \in X$, $y \geq x$, and $y \neq x$ imply that $y \succ x$
- Local non-satiation: The preference relation $\succeq$ on X is locally non-satiated on X iff for any $y \in X$ and $\varepsilon > 0$, there exists $x \in X$ such that $||x - y|| \leq \varepsilon$ and $x \succ y$. (i.e. there are no "thick" indifference curves).

$$\text{Strongly monotone} \Rightarrow \text{Monotone} \Rightarrow \text{Local non-satiated}$$

- Convexity: The preference relation $\succeq$ on X is convex iff the upper contour set is convex (i.e. if $\{x \in X : x \succeq y\}$ is convex). Equivalently, iff $y \succeq x$ and $z \succeq x$ together imply that $\forall \alpha \in [0,1], \alpha y + (1-\alpha)z \succeq x$
- Strict convexity: The preference relation $\succeq$ on X is strictly convex iff $y \succ x$, $z \succ x$, and $y \neq z$ together imply that $\forall \alpha \in (0,1), \alpha y + (1-\alpha)z \succ x$

Exercises

1. Sketch the indifference curves for the following utility functions and say whether preferences represented by these utility functions are strongly monotone, monotone, locally nonsatiated, convex, and strictly convex. Also mention if the utility function is concave or quasiconcave.
   (a) $U(x,y) = x + \alpha y$ where $\alpha > 0$: Strongly monotone, and convex but not strictly convex.
   (b) $U(x,y) = 4 - [(x-2)^2 + (y-3)^2]$ where $U(x,y) \geq 0$: Not locally satiated, strictly convex.
   (c) $U(x,y) = ln(min\{\alpha x, \beta y\})$ where $\alpha, \beta > 0$ and $U(x,y) \geq 0$: monotone but not strongly monotone. Convex but not strictly convex.

*Based on lecture notes and other material by Paul Glewwe. Some examples and exercises come from Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). Microeconomic theory. New York: Oxford university press.

2 Algebra overview

- The $n \times n$ matrix A is **negative semidefinite** if $zAz \leq 0,\ \forall z \in R^n$.
- It is **negative definite** if $zAz < 0,\ \forall z \neq 0$.

Exercises

2. Say whether the following matrix is negative semidefinite and/or positive semidefinite

$$M = \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}$$

Properties:

1. The sum of negative semidefinite matrixes is also negative semidefinite
2. Suppose M is a $n \times n$ matrix, if M is negative semidefinite, then all the diagonal elements must be ≤ 0 (and the inverse is also true).

Let A be a symmetric $n \times n$ matrix. Then we have:

- A is positive definite if $D_k > 0$ for all leading principal minors
- A is negative definite if $(-1)^k D_k > 0$ for all leading principal minors
- A is positive semidefinite if $\Delta_k \geq 0$ for all principal minors
- A is negative semidefinite if $(-1)^k \Delta_k \geq 0$ for all principal minors