APEC 8002: Recitation 3[1]

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Revenue cost function: We can think of the cost minimization problem (under uncertainty) in the following way:

$$\min_{q \geq 0} C(r, q) \qquad s.t. \quad p^s q^s \geq R_s \qquad \forall s = 1, \dots, S$$

The solution to this problem is $q(p, r, R)$, where R is a vector of state-contingent revenues. The revenue cost function is then $C\big(r, q(p, r, R)\big) = C(p, r, R)$.

If FOS(z) satisfies strong free disposal of outputs and $C(p, r^0, R)$ is differentiable at $r = r^0$, we can derive the **state contingent input demands** using the following identity:

$$z_n = \frac{\partial C(p, r^0, R)}{\partial r_n}; \forall n$$

If FOS(z) satisfies strong free disposal of outputs and $C(p, r, R)$ is differentiable at $p = p^0$ and $R = R^0$, we can derive the **state contingent supplies** using the following identity:

$$q_m^s(p^0, r, R^0) = -\frac{\frac{\partial C(p, r^0, R)}{\partial p_m^s}}{\frac{\partial C(p, r^0, R)}{\partial R_s}}; \forall m = 1, \dots, M; \forall s = 1, \dots, S$$

Certainty equivalent revenue: Greatest certain revenue that can be obtained for the same cost at some stochastic revenue

$$ce^R(p, r, R) = \max_e \{e \in \mathbb{R} : C(p, r, e1^S) \leq C(p, r, R)\}$$

Production-risk premium: it measures technological risk.

$$arp^P(\phi, p, r, R) = \bar{R}(\phi, R) - ce^R(p, r, R)$$

where $\bar{R}(\phi, R) = \sum_{s=1}^{S} \phi^s R^S$

Relative production-risk premium:

$$rrp^P(\phi, p, r, R) = \frac{\bar{R}(\phi, R)}{ce^R(p, r, R)}$$

[1] Based on lecture notes

Example:

Consider the cost function

$$c(r, q) = r_1^{2/5} r_2^{3/5} q_1^{1/5} q_2^{\beta/5}$$

Where $r_1 > 0$ and $r_2 > 0$ are input prices; $q_1 \geq 0$ and $q_2 \geq 0$ are outputs, and $\beta > 0$ is a constant parameter. Note that this cost function is derived from a production possibility set that is nonempty, closed, and satisfies weak free disposal of inputs and outputs. Suppose that q_1 represents output in state 1 and q_2 represents output in state 2 of an uncertain world.

a) If p_1 is the price received for output in state 1 and p_2 is the price received for output in state 2, derive the producer's revenue cost function.
b) Derive the state contingent conditional input demands
c) Derive the state contingent supplies using the revenue cost function found in a)
d) Find the certainty equivalent revenue
e) Calculate the production-risk premium and the relative production-risk premium