APEC 8001: Recitation notes 3*

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1 (Quasi)concavity and (Quasi)convexity

- Concavity: A function $f(.)$ is concave if $\forall x, y \in X,\ f[\alpha x + (1-\alpha)y] \geq \alpha f(x) + (1-\alpha)f(y)$. Also if the Hessian matrix of second derivatives is negative semidefinite (i.e. if $zD^2 f(.)z' \leq 0$).
- Quasiconcavity: A function $f(.)$ is quasiconcave if $\forall x, y \in X,\ f[\alpha x + (1-\alpha)y] \geq min\{f(x), f(y)\}$. Also if the Hessian matrix is negative semidefinite subject to a constraint $(\forall z, \bigtriangledown f(x)z = 0)$

$$\text{concavity} \Rightarrow \text{quasiconcavity}$$

$$\succeq \text{ is (strictly) convex iff } u(.)\text{is (strictly) quasi-concave}$$

Exercises

1. Are there any conditions required for $x(p, w)$ to be convex in $p_l = 1, ..., L$?

$$x(p, w) = \frac{w^{a_1+a_2}}{P_1^{a_1} P_2^{a_2}}$$

2 Optimization

The necessary Kuhn Tucker conditions (KKT) are sufficient for optimality if the objective function of a maximization problem is a concave function, the inequality constraints are continuously differentiable convex functions and the equality constraints are affine functions.

$$\max_{x \in \mathbb{R}_+^L} u(x)$$
$$s.t.\ \ px \leq w$$

$\mathcal{L} = u(x) + \lambda[w - px]$

*Based on lecture notes and other material by Paul Glewwe. Some examples and exercises are from Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). Microeconomic theory. New York: Oxford university press.

Necessary conditions:

$$\frac{\partial \mathcal{L}}{\partial x} = \bigtriangledown u(x) - \lambda p \leq 0; \qquad x^*[\bigtriangledown u(x^*) - \lambda p] = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = w - px \geq 0; \qquad \lambda[w - px] = 0$$

F.O.C. is a necessary but not sufficient condition to achieve the maximum. If $u(.)$ is quasiconcave and monotonic, $\bigtriangledown u(x) \neq 0, \ \forall x \in \mathbb{R}^L_+$, then x^* is the solution to UMP.