APEC 8002: Recitation 4[1]

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A producer is **risk averse** with respect to the probability vector ϕ if

$$W(\bar{\Pi}1^s) \geq W(\Pi) \qquad \forall \Pi \qquad where \qquad \bar{\Pi} = \sum_{s=1}^{S} \phi_s \Pi_s$$

$W(\Pi)$ must satisfy the condition that the slopes of the indifference curves evaluated along the equal profits line must be equal to:

$$\phi_s = \frac{\frac{\partial W(1^s)}{\partial \Pi_s}}{\sum_{s'}^{S} \frac{\partial W(1^s)}{\partial \Pi_{s'}}}$$

Certainty equivalent premium: Least amount of certain profit we can give the producer and have it be no worse off than with an uncertain profit

$$ce(\Pi) = \min_c \{c \in \mathbb{R}: W(c1^s) \geq W(\Pi)\}$$

Absolut risk premium: $arp(\phi, \Pi) = \max_c \left\{c \in \mathbb{R}: W\left((\bar{\Pi} - c)1^s\right) \geq W(\Pi)\right\} = \bar{\Pi} - ce(\Pi)$

Relative risk premium: $rrp(\phi, \Pi) = \max_c \left\{c > 0: W\left(\frac{\bar{\Pi}1^s}{c}\right) \geq W(\Pi)\right\} = \frac{\bar{\Pi}}{ce(\Pi)}$

If Π' is a mean preserving spread of Π and $W(\Pi)$ is generalized **Shur-concave**, then:

(i) $\left(\frac{\frac{\partial W(\Pi)}{\Pi_s}}{\phi_s} - \frac{\frac{\partial W(\Pi)}{\Pi_t}}{\phi_t}\right)(\Pi_s - \Pi_t) \leq 0$

(ii) $\sum_{s=1}^{S} \frac{\partial W(\Pi)}{\partial \Pi_s} \left(\Pi_s - \sum_k^S \phi_k \Pi_k\right) \leq 0$

(iii) $ce(\Pi)$ is generalized Shur-concave, such that $ce(\Pi) \geq ce(\Pi')$

(iv) $arp(\phi, \Pi)$ is generalized Shur-convex, such that $arp(\phi, \Pi) \leq arp(\phi, \Pi')$

(v) $rrp(\phi, \Pi)$ is generalized Shur-convex, such that $rrp(\phi, \Pi) \leq rrp(\phi, \Pi')$

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[1] Based on lecture notes

Example

1. Suppose there are only two states of the world, the good state denoted by g and the bad state denoted by b. The producer's profits in the good and bad states are

$$\pi_g = \bar{R}(z) + R(z) - rz \text{ and } \pi_b = \bar{R}(z) - rz$$

Where $\bar{R}(z)$ is certain revenue, $R(z)$ is additional revenue in the good state, $z \geq 0$ is an input, and $r > 0$ is the price per unit of z. If $W(\Pi) = \sum_s \alpha_s \ln \pi_s, \forall s = b, g$

a. What are its subjective probabilities for the good and bad states?
b. What is the certainty equivalent?
c. What is the absolute risk premium? What is the relative risk premium?

2. A farmer can contract his cropland and earn certain net revenue of $r(A) \geq 0$ where A is the number of acres devoted to vegetable production, $r'^{(A)} > 0$ and $r''(A) < 0$. Recently, a bio-refinery is located near the farmer's land and is now offering him a contract to produce switchgrass. The bio-refinery is offering $per ton of switchgrass. The farmer is uncertain about how much switchgrass he can produce per acre. To characterize this uncertainty, assume there are S states of nature and the amount of switchgrass that is produced in the sth state is $q_s \geq 0$ tons per acre. Finally, the cost of growing switchgrass is a constant $c per acre. Let $\bar{A} \geq A \geq 0$ be the amount of land the farmer devotes to producing switchgrass, profits in each state can then be written as $\pi_s = (pq_s - c)A + r(\bar{A} - A)$ for $s = 1, \dots, S$. Assume the farmers preferences, $W(\Pi)$, are generalized Schur-concave with respect to the probability vector $\boldsymbol{\phi} \in \mathbb{R}_+^S$.

   Assuming an interior solution exists, would a farmer with risk neutral preferences probability believes $\boldsymbol{\phi}$ devote more or less land to switchgrass production than a farmer with generalized Schur-concave preferences?