APEC 8001: Recitation notes 5*

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1 Expected utility theory

Von Neumann Morgenstern (vNM) utility function: maps investor?s preference ranking over investments with uncertain money payoffs may be represented by a utility index combining, in the most elementary way (i.e., linearly), the preference ordering on the ex post payoffs and the respective probabilities of these payoffs. $U(F) = \int_0^\infty u(x)dF(x)$

- Certainty Equivalent (CE): Amount of money for which the decision maker is indifferent between the lottery and the certain amount CE
- Risk premium: Difference between the CE and the expected value of the prospect.
- Probability premium: Excess in winning probability over fair (50%) odds that makes the decision maker indifferent between the certain outcome and the lottery between the two outcomes $x+\varepsilon$ and $x-\varepsilon$: $u(x) = (0.5+\pi)u(x+\varepsilon) + (0.5-\pi)u(x-\varepsilon)$

2 Risk aversion

Risk aversion is equivalent to the concavity of $u()$ (the Bernoully utility function).

- Absolute risk aversion: $r_A(x) = -u''(x)/u'(x)$. If positive, then the individual is risk averse. You can also check whether aversion changes across wealth levels. If, for example, r_A decreases, then the Bernoully utility function exhibits decreasing absolute risk aversion (DARA).
- Relative risk aversion: $r_R(x) = -xu''(x)/u'(x)$. Focuses on percentage changes of wealth. If positive, then the individual is risk averse in relative terms. If the coefficient decreases, then the utility function exhibits decreasing relative risk aversion (DRRA).

3 Comparing distributions

Let $F()$ and $G()$ be the cumulative probability distributions or two random variables (cash payoffs).

*Based on lecture notes and other material by Paul Glewwe. Some examples and exercises are from Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). Microeconomic theory. New York: Oxford university press.

- $F(x)$ first-order stochastically dominates $G(x)$ if, for every non-decreasing utility function $u()$ we have: $\int_0^\infty u(x)dF(x) \geq \int_0^\infty u(x)dG(x)$, and $F(x) \leq G(x)$, $\forall x$
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Exercises

1. *Final, Fall 2014*. Behavior under risk. Consider a consumer whose behavior is consistent with the expected utility framework. The consumer has the following Bernoulli utility function: $u(x) = x^{1/2}$
   (a) Suppose the consumer faces the following risky 'lottery': 64 with probability 0.25, 100 with probability 0.50, and 400 with probability 0.25. For this consumer, calculate the certainty equivalent of this lottery. Compare this to the expected income of this lottery.
   (b) Next, consider the same consumer with the same utility function $u(x) = x^{1/2}$. Consider a given amount of money, 25, and another amount of money, 24, that the first amount of money could either increase by or decrease by, each with a probability of 0.50. What is the probability premium of this 'lottery'.
2. *Final, Fall 2013*. This question focuses on behavior under uncertainty for two utility functions that are functions of single aggregate good, denoted by x.
   (a) Suppose that a consumer has a utility function defined by $u(x) = log(x)$. What is the coefficient of absolute risk aversion for this consumer? What is the coefficient of relative risk aversion for this consumer? Is this consumer risk averse? For both of these concepts of risk aversion, does risk aversion increase, stay the same, or decrease as x increases?
   (b) Next, consider another consumer with the utility function $u(x) = x^\alpha$, where $\alpha > 0$. What is the coefficient of absolute risk aversion for this consumer? What is the coefficient of relative risk aversion for this consumer? Is this consumer risk averse? For both of these concepts of risk aversion, does risk aversion increase, stay the same, or decrease as x increases?
   (c) For the two consumers in parts a) and b), is one more risk averse than the other in terms of absolute risk aversion? Explain your answer.
3. *Final, Fall 2015* First and second order stochastic dominance. Consider the following two cumulative distribution functions for income, which is denoted by x:
   - $F(x) = 0$ for $0 \leq x < 1$, $F(x) = 1 - 1/x$ for $x \geq 1$
   - $G(x) = x/10$ for $0 \leq x \leq 10$, $G(x) = 1$ for $x > 10$

   (a) Draw a simple diagram of each of these functions IN SEPARATE DIAGRAMS. You do not need to make a very accurate diagram but just get a rough idea of their shapes. Also, at this point do NOT try to check where one may lie above the other if they were drawn in the same figure.
   (b) Does G(x) first order stochastically dominate (FOSD) F(x)? You should be able to answer this by simply referring to your two diagrams for part a). No mathematics is needed.

(c) Does F(x) first order stochastically dominate (FOSD) G(x)? You will need to use some mathematics for this. [Hint: Find the value of x that yields the minimum or the maximum of the difference between the two functions.]